Benjamin Walter **math210**
Assignment Spring_System_Oscillation due 01/09/2022 at 02:10pm EET

Problem 1. **(1 point)** `METUNCC/Applied_Math/springs/oscillate-theory.pg`

Suppose a spring system with three masses and many springs oscillates at a fundamental mode with

$$\text{eigenvalue } \lambda = 25 \qquad \text{and} \qquad \text{eigenvector } \mathbf{v} = \begin{bmatrix} -5 \\ 5 \\ -1 \end{bmatrix}.$$

(A) If mass 2 oscillates with an amplitude of 2, then what is the amplitude of oscillation for mass 1?
Amplitude = ___

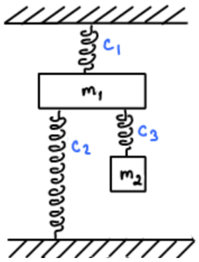
(B) If mass 2 is at maximum height at time $t = 5$, when will it next be at maximum height?
$t =$ ___

(C) Mass 1 oscillates in the [?/same/opposite] direction as mass 3.

(D) If mass 3 begins at displacement $u_3(0) = -4$ and velocity $u_3'(0) = -2$, then what is its dispacement function?
$u_3(t) =$ __________

Problem 2. **(1 point)** `METUNCC/Applied_Math/springs/oscillate-2mass-3springs_alt.pg`

Consider the following spring system.



$c_1 = 1, \quad c_2 = 11, \quad c_3 = 3$
$m_1 = 3, \quad m_2 = 1$

Write the stiffness matrix $K = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

Write the matrix $M^{-1}K = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

Find the eigenvalues and eigenvectors of $M^{-1}K$

- Smaller eigenvalue = ___ with eigenvector $\begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$
- Larger eigenvalue = ___ with eigenvector $\begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$

If this spring system oscillates without any external forces present, then the position of each mass satisfies the following general formula:

$$\mathbf{u}(t) = \left(a_1 \cos(\_\_ t) + b_1 \sin(\_\_ t)\right) \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix} + \left(a_2 \cos(\_\_ t) + b_2 \sin(\_\_ t)\right) \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$$

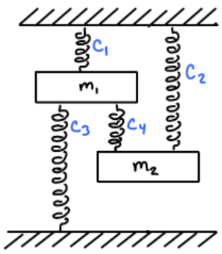
If the system begins oscillation with initial position $\mathbf{u}(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and initial velocity $\mathbf{u}'(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ then the position of the masses at time t is given by

$u_1(t) =$ ______________________________

$u_2(t) =$ ______________________________

Problem 3. **(1 point)** `METUNCC/Applied_Math/springs/oscillate-2mass-4springs_alt.pg`

Consider the following spring system.



$c_1 = \frac{2}{3}, \quad c_2 = 2, \quad c_3 = 0, \quad c_4 = 2$
$m_1 = \frac{2}{3}, \quad m_2 = 2$

Write the stiffness matrix $K = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

Write the matrix $M^{-1}K = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

Find the eigenvalues and eigenvectors of $M^{-1}K$:

- Smaller eigenvalue = ___ with eigenvector $\begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$
- Larger eigenvalue = ___ with eigenvector $\begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$

If this spring system oscillates without any external forces present, then the position of each mass satisfies the following general formula:

$$\mathbf{u}(t) = \Big(a_1 \cos(\_\_\, t) + b_1 \sin(\_\_\, t)\Big) \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix} + \Big(a_2 \cos(\_\_\, t) + b_2 \sin(\_\_\, t)\Big) \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$$

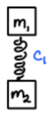
If the system begins oscillation with initial position $\mathbf{u}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and initial velocity $\mathbf{u}'(0) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ then the position of the masses at time t is given by

$u_1(t) =$ ______________________

$u_2(t) =$ ______________________

Problem 4. **(1 point)** `METUNCC/Applied_Math/springs/oscillate-detatched.pg`

Consider the following unattached spring system.



$c_1 = 2,$
$m_1 = 1, \;\; m_2 = 1$

Write the stiffness matrix $K = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

Write the matrix $M^{-1}K = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

Find the eigenvalues and eigenvectors of $M^{-1}K$:

- Smaller eigenvalue = __ with eigenvector $\begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$
- Larger eigenvalue = __ with eigenvector $\begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$

If the spring system oscillates beginning with initial displacement $\mathbf{u}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and initial velocity $\mathbf{u}'(0) = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$ then compute the displacements of the masses at time t.

$u_1(t) =$ ________________
$u_2(t) =$ ________________

Find a nonzero initial velocity vector such that the displacement of the masses will be bounded.

- $\mathbf{u}'(0) = \begin{bmatrix} \text{\_\_\_} \\ \text{\_\_\_} \end{bmatrix}$